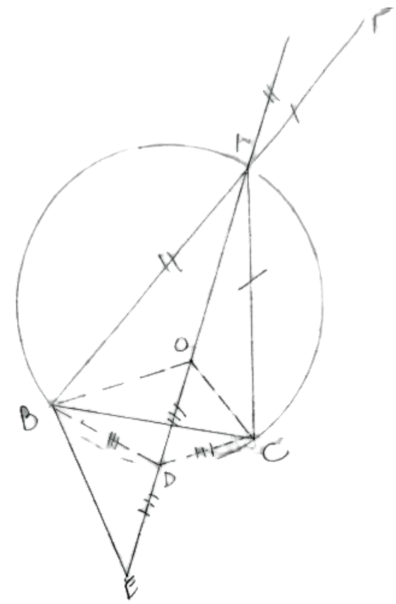
# 01.03.2024 - Cash Award Math Rider II Prize Winner Mr.Peeyush Rajan Panda's Solution




**Given :**

AC= AF

OA = AG

OD = DE

O is incentre of $\Delta ABC$

**Construction :**

Join OC, OB, BD, DC

**Proof:**

$\angle BAO = \angle OAC$ (O is incentre. AO angle bisector)

$\angle BAO = \angle GAF$ (Vertically opposite)

$\therefore \angle OAC = \angle GAF$

In $\Delta\, GAF, \Delta OAC$

GA = AO ( given)

AF = AC (given)

$\angle GAF = \angle OAC$ (derived earlier)

$\therefore \Delta GAF \cong \Delta OAC$

$\angle ACO = \angle AFG$ -------------- (1)

$\angle ACO = \angle BCO$ (O is incertre. CO is bisector)

$\angle BCD = \angle BAD$ (same segment)

$= \angle DAC$ (O is incentre)

$\therefore \angle OCD = \angle OCB + \angle$BCD

$= \angle ACO + \angle DAC$

$= \angle ACO + \angle OAC$

Now, $\angle DOC = \angle OAC + \angle OCA$ (Exterior sum of interior opposite angle)

$\therefore \angle DOC = \angle OCD$

⇒ DOC isosceles triangle

OD = CD

$\angle DAC = \angle DAB$ ( O in centre)

⇒ DC = DB (equal chord subtend equal angle)

∴ DB = OD

DB = DE (given)

∴OD = BD = ED

median is equal to half of side length.

$\therefore \angle OBE = 90$

$\angle ACB = \angle ADB$ (same segment)

BD = DE

$\therefore \angle BDA = \angle DBE + \angle DEB$

$= 2\,\angle DEB$ (BD = DE $\Rightarrow \angle DEB = \angle DBE$)

$\Rightarrow \angle DEB = \frac{1}{2}\angle BDA$

$= \frac{1}{2}\angle ACB$

$= \angle ACO$ ------------- (2) (O is incentre CO bisector)

From (1) , (2)

$\angle ACO = \angle DEB = \angle AFG$

$\Rightarrow \angle GEB = \angle GFB$

∴ GFEB quadrilateral is cyclic.

(because angles in same segment are equal)

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